Supplementary Appendix to Largest-Index Zonotope Bounds for Planar Projections of Semiorder Ideals

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1 Purpose and evidentiary boundary

This appendix records the exact computation, reconstruction provenance, claim-to-code map, replay protocol, and negative-result boundaries for the accompanying theorem note. The main theorem is proved in the manuscript. Every computation described here is finite falsification or regression evidence. No checker output is treated as a proof, no finite census is promoted to an infinite theorem, and no internal reconstruction is described as external peer review.

2 Frozen claim matrix

The verifier uses the inclusive represented-semiorder convention

$$x_i \le_P x_j \iff u_i \le u_j - \theta, \qquad \theta > 0,$$

for distinct elements sorted by nondecreasing utility with a fixed-label tie break. Element vectors lie in $\mathbb{Q}^2$ and projected ideals are deduplicated before strict hull vertices are counted.

Not promoted are optimal constants, linearity for unrestricted semiorders, results for arbitrary posets, historical priority, exact finite values of a global extremal function, or an epistemic repair application.

3 Dependency and falsifier map

The proof has six load-bearing nodes:

D1. the largest-index fiber identity $F_j \cup \{j\} \cup T$, $T \subseteq W_j$;

D2. convexification of each fiber to $t_j + \sum_{i \in W_j} [0, v_i]$;

D3. the local planar zonotope bounds $q_j + 1$ and $\max\{1, 2q_j\}$;

D4. inheritance of strict global vertices from the origin or a constituent;

D5. the exact charge $\sum_j q_j = \mathrm{inc}(P)$;

D6. the antichain window bound $q_j \le \min\{j, r-1\}$ and its exact sum.

Table 1: Promoted statements and the role of computation.

Identifier	Exact statement	Evidence role
C-SEMI-U	$h(P, v) \le 1 + n + \operatorname{inc}(P)$	Written proof; verifier attacks all steps.
C-SEMI-F	$f(P, v) \le 1 + \sum_j \max\{1, 2q_j\} \le 1 + n + 2\operatorname{inc}(P)$	Written proof; exact full-hull audit.
C-SEMI-WU	$h(P, v) \le 1 + rn - \binom{r}{2}$ for $r = \min(n, w)$	Written summation; width recomputed exactly.
C-SEMI-WF	$f(P, v) \le n + 1$ for $r = 1$, and $f(P, v) \le 2 + (r - 1)(2n - r)$ for $r \ge 2$	Written local-sum optimization; verifier checks boundary cases.
C-SEMI-L	A mixed-sign chain has $n + 1$ strict upper and full vertices	Exact construction; symbolic and finite checks.

The cheapest mathematical falsifier is a represented semiorder and rational coefficient assignment violating any one node. High-value hostile cases are threshold equality, duplicate utilities, empty and singleton posets, all-zero coefficients, vertical-only systems, parallel and antiparallel generators, simultaneous support roots, duplicate projections, one-dimensional hulls, and upper endpoints. The cheapest source-status falsifier is a primary source that states or immediately implies the same incomparable-pair or $O(nr)$ projection bound.

4 Standalone verifier

The decisive program is `independent_checker/verify_semiorder_geometric_bound.py`. It uses only Python integers and `fractions.Fraction`. It imports no promoted optimization engine, certificate builder, semiorder theorem module, or originating proof implementation.

For each case it independently:

1. reconstructs the relation from utilities and threshold;
2. enumerates all ideals and exact projections;
3. computes strict upper and full convex hulls by integer cross products;
4. reconstructs the largest-index fibers and their local hulls;
5. counts incomparable pairs and width;
6. checks local vertex bounds and global vertex inheritance;
7. checks every displayed instance-sensitive and width-sensitive bound.

This is method-distinct internal checking. Because it was produced in the same autonomous research program, it is not external independent review.

5 Exact finite scope

Table 2: Frozen verifier scope and result.

Field	Exact value
Status	PASS
Total exact cases	318,667
Ideals enumerated	3,978,057
Distinct projected points	2,311,018
Fiber points enumerated	3,019,505
Global upper vertices	879,270
Global full vertices	1,487,576
Cases with tied utilities	315,659
Cases with threshold equality	104,494
Cases with simultaneous support roots	80,678
Cases with zero vectors	117,753
Cases with vertical nonzero vectors	199,380
Cases with parallel generators	148,710
Cases with antiparallel generators	138,032
Maximum n in adversarial fixtures	9
Maximum width observed	8
Result-core SHA-256	`2f5da66840fb3a837d022ec1a48499b2` `4df659dc6cefdb82c2166e62517891e8`

The exhaustive portion covers all nondecreasing utility tuples with repetition from $\{0, 1, 2\}$, thresholds $1, 2, 3$, and all ordered vector assignments from a nine-vector alphabet, for the stated sizes. Ten additional adversarial fixtures isolate degeneracies and larger fibers. This finite scope is deliberately reported exactly rather than described as exhaustive over all semiorders or coefficients.

Two older isolated audit lanes supply additional non-load-bearing evidence: 245,157 generator multisets through seven listed generators; 14,000 global upper-hull cases; 10,000 global full-hull cases; and 12,051 end-to-end exact rational cases. Their reports and hashes are preserved in the package.

6 Fresh clean reconstruction

The publication audit reconstructed the source repository from the preserved Git bundle rather than relying on a mutable working directory. The relevant identifiers are:

- repository bundle SHA-256:
  `47cf205b7be3b843fa5797ba28d919d3`
  `e5e18ef0a9ad7d9d0f528b59ef0fc89b`;
- annotated release tag: `v0.1.0-semiorder-width-bound`;
- theorem release commit: `d8c3b8a3fbd960f70e7f6eb1634a62731c2e0a40`;
- clean-replay commit: `f9f0edf1d853a6de7b3c1e7a70076748a49a52bc`;

- standalone verifier SHA-256:
  `b68e3ef7a43c23b29ba2a8ea8883b7`
  `d4488f666315d25cbf10ddc7eb1fd17e96`.

The source gate matched all ten required local source objects and the canonical manifest digest `591a181de3299ab54bb183e7ec2fe0a0ece7dc32e69f72cc181e2dcf4492c852`. The machine manifest and replay ledger contain the same authoritative digest and must be used for automated comparison.

The exact replay command is

```
python3 -B independent_checker/verify_semiorder_geometric_bound.py
```

It returned PASS on 318,667 cases, 3,978,057 ideals, and 2,311,018 distinct projected points, with result-core SHA-256 `2f5da66840fb3a837d022ec1a48499b24df659dc6cefdb82c2166e62517891e8`. The August 2026 replay required 47.12 seconds of wall time and no nonstandard Python dependency.

7 Source and prior-art audit

The source manifest includes Eppstein's author manuscript for *The Parametric Closure Problem* and Wood's paper on clique counts. Their local PDF SHA-256 values are, respectively,

```
44dda77a9511a72ab795d62ceb9b13795b40f4e39fc336340eb149c72421cde9
45b3a8c1c4e391c3a24f8d79b9a080d78849e5ee619ecc43f0284144ac872ecf
```

The primary-source comparison verifies that Eppstein proves the general $O(n \log n)$ full-polygon bound for semiorders, while Wood's clique theorem subsumes the abandoned ideal-count route. A refreshed bounded search in August 2026 used exact formulas, equivalent terminology, semiorder and unit interval order terminology, order-polytope shadows, supported ideals, bicriterion closure, citation descendants, and software queries. No exact geometric collision was identified. The correct disposition remains `UNRESOLVED_AFTER_BOUNDED_SEARCH`; bounded search does not prove novelty or absence.

8 Corrections and negative results

The theorem release followed seven recorded correction transactions covering an oracle-hull invariant, recursion depth, certificate trust, atomic publication, large-certificate scope, realization input validation, and clean-checkout import paths. Those corrections were closed before the tagged theorem release.

The broader unrestricted campaign later established several useful non-results that are not dependencies of this paper:

- additive series and parallel compositions cannot by themselves yield a superlinear construction;
- a finite product-grid recurrence failed at its next member;
- a coordinate hill climber under-reports even a known four-element fixture, so its curves are lower-bound search evidence only;
- the order-polytope adjacency core of a transition route is prior art;

- an oriented-path sign-change bound is weaker than the known asymptotic bound without an additional uniform estimate;
- finite values such as $h = 7$ or $h = 9$ for individual assignments are not exact extremal values or asymptotic evidence.

These boundaries prevent the publication package from implying an unrestricted parametric-closure result or splitting implementation infrastructure into an artificial second paper.

9 Artifact and replay checklist

The reproducibility directory includes:

- this exact verifier and its frozen expected JSON;
- frozen proof, source-audit, independent-audit, and clean-checkout records;
- a standard-library replay driver and unit tests;
- environment and dependency locks;
- input and output hashes, a data dictionary, and artifact inventory;
- a no-grant license-status notice recording that default copyright applies;
- SHA-256 manifests and known limitations.

The replay succeeds only if the verifier status, exact counts, verifier digest, and canonical result-core digest all match. A nonzero exit, schema change, count difference, hash difference, or unexpected file mutation is a failure; it must not be normalized to PASS.

10 Disclosure boundary

Automated systems assisted theorem development, implementation, testing, source organization, and writing. They are neither authors nor external reviewers. Final author identity, affiliation, correspondence, funding, competing interests, authorship approval, and any later license assignment require human confirmation. The public GitHub preprint release is separate from journal or arXiv submission; this appendix performs no submission or email action.